THE RECITATION IN MATHEMATICS.

The purpose of education is to help the individual better to meet both present and future life situations. The process by which this purpose is realized continues throughout life and the means of education includes every phase of our environment which tends to modify the way in which we react to a given situation. Clearly the school is only one of the means of education. Within the school various forms of student activity are rightfully assuming an ever increasing importance, but at present most teachers look upon the recitation as the most important educative factor in the school. The function of the recitation seems to be to pass on to the next generation an accumulation of experience which school authorities believe to be essential to the welfare of society and in so doing to develop certain desirable abilities and capacities in the individual. Unfortunately in our attempt to realize this function we have separated our subject matter from its useful relationships and in the child's mind it is a mass of material almost wholly unrelated to the world in which he lives, and our very attempts have defeated our purpose. This paper discusses certain fundamental principles which will compel a closer relationship between mathematics and out-of-school life and will therefore make the recitation a more effective means of realizing the aim of education.

Nothing is of educational value which does not result in some form of activity on the part of the one being educated. Not only physical but also mental and emotional activities are here included. This principle denies the value of a recitation in which the pupils are passively receptive and the teacher is the active agent. We may explain to the child how to ride a bicycle but it is only by trying that he learns. The teacher may develop mathematical theorems, solve problems, and explain processes all to little or no purpose as long as his class remains passive. Ready made proofs of our geometry texts require little or no reasoning on the part of children and there-

fore, of themselves, cannot possibly develop the power to reason. This principle demands that the pupil as well as the teacher shall be actively engaged in the work of the recitation. This does not mean that the teacher is to be eliminated. He is to stimulate and guide the activities of the pupils, but if the recitation is to be effective, they must do the work.

"*Practical skill, modes of effective technique, can be intelligently, non-mechanically used, only when intelligence has played a part in their acquisition.*"* It has been said that all the geometry that is needed for practical purposes can be taught in a very few lessons. The authors of such statements undoubtedly mean that the facts which find frequent applications in the lives of most individuals can be taught in a mechanical rule-of-thumb way in a few lessons. Such teaching, however, is a violation of the principle under discussion. If facts are thus taught without an intelligent understanding of the underlying principles, the pupil may be able to use them under the exact conditions under which they were learned, but when application is to be made to a new situation it is doubtful if he can succeed. By mere imitation we can, in a very short time, teach a pupil to bisect a line by the usual method of constructing two intersecting arcs on either side of the line and connecting the two intersections. The chances are that he will then fail to bisect a given length laid off along the intersection of the floor and wall. If, however, he has been taught that *just any two points* equidistant from the extremities of the line determine the perpendicular bisector of the line, the chances of his solving the situation are much greater. In the problem $3\underline{)948}$, the child is taught in a mechanical way that the remainder 1 obtained from dividing 4 by 3 together with 8 make 18. Later, even in high school, in the problem $3\underline{)964\frac{3}{8}}$ he assumes that the remainder 1 and 3 make 13, and he consequently divides $1\frac{3}{8}$ instead of $1\frac{1}{8}$ by 3. If, in the first case, he had been taught that he was reducing from one denomination to another, he would, in the second case, more probably have reduced the 1 to eights. Examples can be multiplied, but it is clear that if mathematical education is to be most useful, there must be an intelligent mastery of facts. The rule-of-thumb method must be replaced by a rational method.

* Dewey, John, "How We Think," p. 52.

Those and only those processes which will occur frequently in the lives of all should, through drill, be reduced to habit. There is a tendency on the part of some educators to eliminate all drill from the recitation. On the other hand, there are many, especially among mathematics teachers, who devote a majority of the time to drill. The principle safeguards against both of these extremes. It at once provides for drill on fundamentals and demands that all drill on non-essential and on essential material which is only occasionally useful shall be eliminated. Efficiency demands that the number combinations, the laws of signs, the laws of exponents, and the association of certain properties with the parallelograms shall become a matter of habit. On the other hand in algebra pupils are drilled on long division, forms of factoring, difficult fractions, and complicated fractional equations none of which can by any chance be of value to the majority of pupils. In trigonometry there are difficult formulæ which are necessary but which will occur only occasionally in the lives of most people. Pupils are compelled to memorize them although they forget them immediately after examinations. Instead of memorizing such material, the time should be spent on learning how to use it. When the need arises the pupil can look it up, which he will have to do any way as he will have long since forgotten it. Just as surely as efficiency demands that certain processes shall be reduced to habit, it also demands that we shall not drill on material which will not occur frequently in the lives of nearly all people.

Only those things which have been learned in useful relationships will surely be useful. The fact that a pupil has learned certain phases of pure mathematics does not mean that he will recognize its usefulness in the solution of problems outside of the class-room. Students who can solve quadratic equations in the algebra class fail to use them in the physics class. Only recently a student who had completed the study of plane and solid geometry, failed completely to recognize a geometrical situation in the question, "How can a sailor determine how many degrees of latitude he is north of the equator?" A student of calculus who could easily find the value of x which gives a maximum for a function of x, never so much

as associated calculus with the problem, "How large must be the squares cut from the corners of a rectangular piece of cardboard 10 inches by 20 inches so that a box of the largest possible capacity may be made? If, however, we should begin with practical problems and let the formal phases of mathematics grow out of them, and if we should follow this up with a variety of applications and encourage our pupils to look for other applications, mathematics will become a useful tool for the solution of practical problems whenever they are met.

Dr. Thorndike has said,* "Teach nothing merely because of its disciplinary value, but teach everything so as to get what disciplinary value it does have." *The best way to get what disciplinary value mathematics has is to get the greatest possible practical value it has.*† Perhaps geometry has been defended more strongly as a means of developing the ability to think than any other subject. However, if we examine the text-book method by which geometry is usually taught we find that almost no element of the thought process is present. Dewey‡ analyzes the act of thought into the following five steps, "(i) a felt difficulty; (ii) its location and definition; (iii) suggestion of possible solution; (iv) development by reasoning of bearings of the suggestion; (v) further observation and experiment leading to its acceptance or rejection; that is the conclusion of belief or disbelief." Ordinarily no one of these steps is present in the text-book presentation of geometry. The student does not feel a real purpose in proving a theorem; he does not define the problem or state what is to be done. Neither does he propose and develop a possible solution nor does he draw a conclusion. In fact all of these are done for him and he too frequently memorizes the text and *recites* it to the teacher. If the thought process is not present, then surely the ability to think can not be developed.

However, let us undertake to lay off a rectangular field for a Maypole dance. After the pupils have established one side (i) they have the problem of turning a square corner; (ii) they define it as erecting a perpendicular to a line at a given point in the line; (iii) they suggest various methods of pro-

* Thorndike, E. L., "Principles of Education," p. 249.

† Mair, David, "School Course in Mathematics," p. iii.

‡ Dewey, John, "How We Think," pp. 68–72.

cedure; (iv) they consider each of these in the light of every available fact; and (v) those suggestions that give promise are carefully investigated, proved or disproved, and finally accepted or rejected. Such a process involves a complete act of thought and therefore will develop in the fullest possible way the ability to think; and it has the additional advantage of teaching the pupil to use his geometry outside the school room.

It is not possible to give a complete discussion of the mathematics recitation within the limits of this brief paper. However, we believe that the five fundamental principles stated above provide for almost all valuable phases of the recitation. If they are considered in the conduct of the recitation, there must be a conscious purpose on the part of the pupil; facts must be related to life outside the school room; we must provide for the mastery of important mathematical truths; drill on fundamentals is essential; student activity calling for co-operation within the class-room and in field work provides an opportunity for the socialized recitation; and finally useful activity requires the problem method of teaching.

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